

A Tale of Two Distances

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There are two basic ways to measure physical distances in cosmology: One based on standard candles and one based on standard rulers. Comparing current data for each method allows us to rule out axion-photon mixing and dust-extinction as the sources of supernova dimming and generally protects the case for cosmic acceleration from attacks based on loss of photons. The combined data constrains the energy densities in a Λ CDM model to $0.19 < \Omega_m < 0.32$ and $0.47 < \Omega_\Lambda < 0.82$ (at 2σ) without recourse to any further data sets. Future data will improve on these limits and allow us to place constraints on more exotic physics.

1 Introduction

The concept of absolute space and time disappeared in the transition from the Newtonian theory of gravitation to General Relativity. Nonetheless, there is a general duality in any metric theory of gravity implying that distances in cosmology are unique^{1,2,3}. The luminosity distance ($d_L(z)$, based on the apparent luminosity of standard candles) and the angular-diameter distance ($d_A(z)$, based on the apparent size of standard rulers) are linked by distance-duality:

$$\frac{d_L(z)}{d_A(z)(1+z)^2} = 1. \quad (1)$$

where z is redshift. Distance-duality holds for general metric theories of gravity in any background (not just Friedmann-Lemaître-Robertson-Walker [FLRW]) in which photons travel on unique null geodesics and is essentially equivalent to Liouville's theorem in kinetic theory. It is only valid if photons are conserved, but is not violated by gravitational lensing (for infinitesimal geodesic bundles).

In these proceedings we will start by comparing the two distances in the standard FLRW framework. To show the power of distance-duality as a test of non-standard physics, we then

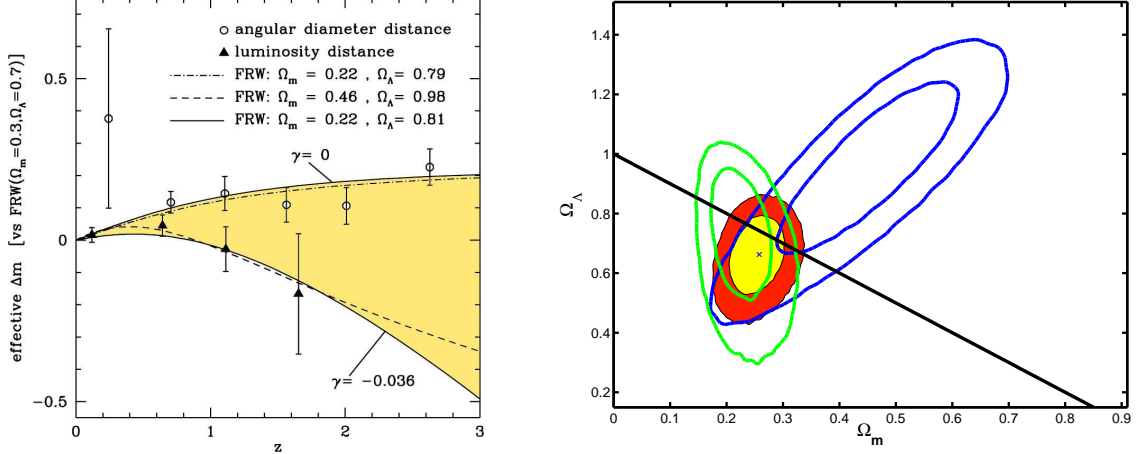


Figure 1: Left panel: The binned data for $d_L(z)$ (triangles, SN-Ia) and $d_A(z)$ (circles) are shown in equivalent magnitudes relative to the flat concordance model ($\Omega_\Lambda = 0.7, \Omega_m = 0.3$) with 1σ error bars. They should coincide if distance-duality holds. The dashed curves are the best-fit FLRW models to the $d_A(z)$ data (top) and $d_L(z)$ (bottom) separately with no loss of photons. The solid curves have the same underlying FLRW model ($\Omega_\Lambda = 0.81, \Omega_m = 0.22$) but the lower curve includes the best-fit exponential brightening (see Bassett & Kunz⁴ for details). Right panel: 1 and 2σ likelihood contours for Ω_m and Ω_Λ in a Λ CDM framework for the angular diameter distance data alone (green/light-grey), the luminosity distance data alone (blue/dark-grey) and the combined data sets (filled contours).

use it to rule out replenishing dust as the source of the supernova dimming and to constrain axion oscillations over cosmological distances. More details can be found in^{4,5}.

Our estimates of the luminosity distance $d_L(z)$ is provided by the latest compilation of type-Ia supernova data⁶. This data set includes a significant number of $z > 1$ observations. Our angular-diameter distance data, $d_A(z)$, come from FRIIb radio galaxies^{7,8}, compact radio sources^{9,10,11} and X-ray clusters¹². It is important to remember that some of this data predated the discovery of acceleration by SN-Ia and that there are now completely independent, indirect, estimates of d_A , e.g. from analysis of the 2QZ quasar survey¹³ (giving $\Omega_\Lambda = 0.71^{+0.09}_{-0.17}$) and strong lensing from a combination of the CLASS and SDSS surveys with a maximum likelihood value of $\Omega_\Lambda = 0.74 - 0.78$ ¹⁴, in good agreement with estimates from radio sources. The Sunyaev-Zel'dovich effect in galaxy clusters is a further possible source of distance data^{15,16}.

2 Applications of distance-duality

2.1 Comparison within standard cosmology

As a first test, we plot the binned data as a function of redshift in figure 1. We show equivalent magnitudes relative to the flat concordance model ($\Omega_\Lambda = 0.7, \Omega_m = 0.3$) with 1σ error bars. Although the supernova data lies systematically below the angular diameter distance data (and is thus *too bright*), the violation of the distance duality is only at the 2σ level and thus not significant enough to claim a detection^{4,16}.

Keeping this possible violation of the duality relation in mind, we will nonetheless combine the two data sets to derive limits within two frameworks: In the first one we assume a Λ CDM cosmology while in the second one we restrict ourselves to flat universes, but let the equation of state parameter $w = p/\rho$ of the dark energy component vary (although we assume it to be constant). The right hand panel of fig. 1 shows the 1 and 2σ contours in the $(\Omega_m, \Omega_\Lambda)$ plane for the SN-Ia data alone (blue/dark-grey contours), the angular diameter data alone (green/light-grey contours) and the combined data (filled contours). The diagonal black line shows the flat

Table 1: Limits on the cosmological parameters obtained by combining current $d_L(z)$ and $d_A(z)$ data (the Hubble constant is always being marginalised over).

| data set | Λ CDM | | flat universe | |
|----------|-----------------|------------------|-----------------|-------------------------|
| | Ω_m | Ω_Λ | Ω_m | w (95%CL) |
| $d_L(z)$ | 0.45 ± 0.11 | 0.94 ± 0.19 | 0.50 ± 0.06 | $-3.86^{+2.73}_{-6.36}$ |
| $d_A(z)$ | 0.23 ± 0.04 | 0.70 ± 0.15 | 0.22 ± 0.05 | $-1.00^{+0.40}_{-0.45}$ |
| combined | 0.25 ± 0.03 | 0.66 ± 0.09 | 0.25 ± 0.05 | $-0.94^{+0.28}_{-0.37}$ |

models. The marginalised limits on the cosmological parameters are given in table 1. In the second case, the supernovae alone yield only very weak constraints on w without additional constraints on Ω_m . The angular diameter distance data is less susceptible to this problem due to the larger redshift range, and the combined data requires $-1.31 < w < -0.66$ at 2σ .

Even the combined data sets are unable to constrain all three parameters ($\Omega_m, \Omega_\Lambda, w$) simultaneously, as universes with low Ω_m and Ω_Λ together with a very negative equation of state provide a good fit to all the data. Only the matter density can be constrained in this case, $0.15 < \Omega_m < 0.31$ (95% CL).

2.2 Ruling out replenishing dust

Riess *et al*⁶ found that the best-fit model to all currently available SN-Ia was not an accelerating Λ CDM model but rather a replenishing grey-dust model¹⁷ with $\Lambda = 0$ which causes redshift-dependent dimming of the SN-Ia, with the evolution of ρ_{dust} changing from $\propto (1+z)^3$ to a constant at $z = 0.5$. If this was the correct explanation then we should expect a marked violation of distance duality with the d_A data lying below the d_L data since it would correspond to a non-accelerating universe. Our results show that this is not the case (indeed we have the opposite problem!)

A detailed analysis of this model based on^{17,18} gives a best-fit to *all* the data of $\Omega_\Lambda = 0.77 \pm 0.13$ showing that the combined data, in contrast to the SN-Ia data alone, rule out the replenishing dust model at over 4σ .

2.3 Limits on axion oscillations

Another mechanism that was recently proposed¹⁹ explains the supernova dimming (relative to a $\Lambda = 0$ cosmology) by allowing photons to oscillate into axion states. In this way, about a third of the photons are lost over cosmological distances. Again, as in the case of dust, the angular diameter distance is unaffected, and should thus correspond to the one expected for a standard CDM universe. As fig. 1 shows, this is absolutely not the case, and axion-photon mixing cannot explain away the need for a dark energy component.

We analysed this case in more detail⁵ by modeling the transition probability as

$$P_{\gamma \rightarrow \gamma} = \frac{2}{3} + \frac{1}{3} e^{-l/l_{\text{dec}}} \quad (2)$$

and by introducing the dimensionless damping amplitude $\lambda \equiv 1/(2H_0 l_{\text{dec}})$ which is zero if no mixing occurs and one in the case of mixing over cosmological distances. The combination of luminosity and angular diameter distance data limits the mixing to $-0.7 < \lambda < 0.3$ and the equation of state parameter of the dark energy component to $-1.6 < w < -0.6$, both at 2σ .

A priori the absence of observed oscillations could be used to place stringent constraints on the axion-photon coupling. If we require the decay length l_{dec} to be of the order of the Hubble scale and follow¹⁹, we end up with an upper limit of $g_{a\gamma} \sim 1/M \sim 2 \times 10^{-12}/\text{GeV}$. But this

limit holds only for ultra-light axions, $m_a < 10^{-14}$ eV. The oscillation probability for heavier axions is suppressed by a factor proportional to $1/m_a^4$.

3 Discussion and conclusions

We have shown that distance-duality is a powerful tool for constraining a variety of modifications of standard cosmology as well as for improving our knowledge of cosmological parameters. In particular, we are able to constrain the energy densities in a Λ CDM universe at the 95% confidence level to $0.19 < \Omega_m < 0.32$ and $0.47 < \Omega_\Lambda < 0.82$ purely based on the expansion history of the universe. The case $\Lambda = 0$ is ruled out at very high confidence. This result does *not involve any perturbations* and is thus not affected by issues like the initial spectrum of perturbations or uncertainties in the determination of $\Omega_b h^2$ and the reionisation optical depth.

We have further shown that there is no evidence for any strong attenuation of high-redshift supernovae by dust (even dust tailored to mimic the expansion-induced dimming) or for the loss of photons due to axion-photon mixing.

With future experiments like the JDEM/SNAP satellite mission²⁰ and the KAOS/gwfmms galaxy survey²¹, we expect to test deviations from distance duality at the level of a few percent, implying that this diagnostic will mature into a unique and powerful test of fundamental physics on cosmological scales.

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